Geographical diversification in wheat farming: a copula-based CVaR framework

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Abstract

Purpose – Portfolio theory suggests that geographical diversification of production units could potentially help manage the risks associated with farming, yet little research has been done to evaluate the effectiveness of a geographical diversification strategy in agriculture. The paper aims to discuss this issue.

Design/methodology/approach – The paper utilizes several tools from modern finance theory, including Conditional Value-at-Risk (CVaR) and copulas, to construct a model for the evaluation of a diversification strategy. The proposed model – the copula-based mean-CVaR model – is then applied to the producer's acreage allocation problem to examine the potential benefits of risk reduction from a geographical diversification strategy in US wheat farming. Along with the copula-based model, the multivariate-normal mean-CVaR model is also estimated as a benchmark.

Findings – The mean-CVaR optimization results suggest that geographical diversification is a viable risk management strategy from a farm's profit margin perspective. In addition, the copula-based model appears more appropriate than the traditional multivariate-normal model for conservative agricultural producers who are concerned with the extreme losses of farm profitability in that the later model tends to underestimate the minimum level of risk faced by the producers for a given level of profitability.

Originality/value – The effectiveness of geographical diversification in US wheat farming is evaluated. As a methodological contribution, the copula approach is used to model the joint distribution of profit margins and CVaR is employed as a measure of downside risk.

Keywords Agriculture, Copulas, Portfolio theory, Conditional Value-at-Risk,
Geographical diversification

Paper type Research paper

1. Introduction and background

An agricultural producer faces many types of risk, including fluctuations in yields and prices. Moreover, a changing set of government policies can cause wide swings in farm profitability (i.e. the ability of a farm to generate profit). Due to different soil types, different weather variables (temperature, rainfall, etc.), and many other factors, the profitability of farming varies across states and regions over time (Nartea and Barry, 1994; Krueger *et al.*, 2002; Blank *et al.*, 2005). Through less-than-unit correlations among regional profitability, portfolio theory suggests that geographical diversification of production units could potentially help manage the risks associated with farming. In fact, some farms have already begun to diversify geographically (Nartea and Barry,

1994). Little is known, however, about the effectiveness of a geographical diversification strategy in agriculture.

Previous studies of geographical diversification have produced somewhat contradictory results. For example, Nartea and Barry (1994) analyzed the costs and returns of geographical diversification in Central Illinois to determine whether geographical diversification was a legitimate risk management strategy for individual grain growers. By comparing the increases in revenues received with increases in transportation and monitoring costs and losses due to poor machinery coordination, the author concluded that there was no realizable gain from diversifying geographically in Central Illinois. Davis *et al.* (1997), on the other hand, examined the impact of geographical diversification on peach orchards in Georgia. They argued that weather related production risks in peach orchards could be reduced through spatial scattering. In particular, using a stochastic production function, the authors determined the variability of yield that could be reduced by geographically scattering peach orchards. They found that for every mile increase in the distance between orchards, a correlation between yields dropped by 2 percent. They then concluded that implementing geographical diversification was a legitimate risk reduction strategy and that geographical diversification could also enhance the long-term sustainability of peach production. Later, Krueger *et al.* (2002) analyzed the effects of an international geographical diversification strategy on a producer's profit margin. Their analysis was applied to the California-Chile table grape industry. The results from a firm-level simulation model showed that geographical diversification could be profitable for a US table grape producer. Further, Oglend and Tveteras (2009) evaluated the effectiveness of spatial diversification in Norwegian salmon aquaculture. Using the standard Markowitz's (1952) mean-variance portfolio analysis, they found that geographical diversification could significantly reduce variations in the rate of returns. They thus concluded that geographical diversification was a viable risk management strategy in salmon farming.

This paper adds to the scarce literature on geographical diversification in agriculture by examining the effectiveness of a geographical diversification strategy in wheat production in three separate locations: Smith County, Kansas; Bottineau County, North Dakota; and Dumas in Moore County, Texas. The three locations are chosen based on harvesting windows and distance criteria. Texas and Kansas are winter wheat production regions while North Dakota is a spring wheat production region. The typical planting and harvesting sequence would begin in Texas and end in North Dakota. In theory, this would allow a producer to use the same machinery set in all three regions. Similar to Oglend and Tveteras (2009), the paper applies portfolio theory to production allocation decisions to find profit maximizing (or risk minimizing) outcomes and to evaluate the effectiveness of a geographical diversification strategy. However, instead of using variance (or, equivalently, standard deviation) as a risk measure, this paper employs one of the recent most popular risk metrics in financial literature – Conditional Value-at-Risk (CVaR), which is a downside risk measure. The application of CVaR as a risk measure addresses two major drawbacks of the traditional mean-variance optimization approach. First, when variance is used as the risk measure, all risk is treated the same (i.e. upside risk is penalized the same as downside risk). This symmetric view of uncertainty is counter-intuitive because upside risk is often considered to be the riskless opportunities for unexpected high returns (Alexander and Baptista, 2004). Because individuals, as well as agricultural producers, are generally concerned only with the downside risk, CVaR is used as a measure of downside risk in this study. Second, variance is a valid risk measure only when asset returns in the portfolio are normally distributed (Szegö, 2005). However, in reality asset returns, as well

as agricultural prices and yields, have been shown to be non-normal (Goodwin and Ker, 2002; Ramirez *et al.*, 2003; Sun *et al.*, 2009). Thus, CVaR has an advantage over variance in that it does not rely on the assumption of multivariate normality. This study, therefore, adopts the mean-CVaR framework in analyzing potential benefits from geographical diversification. Another popular downside risk measure, Value-at-Risk (VaR), is not chosen because VaR has been found to be a less consistent measure of downside risk than CVaR (Artzner *et al.*, 1999; Rockafellar and Uryasev, 2000; Alexander and Baptista, 2004).

Specifically, the study develops a portfolio of wheat production locations that maximizes farm profitability (measured by a profit margin from production) for a given level of CVaR. The mean-CVaR framework requires knowledge of the joint distribution of profit margins in the calculation of CVaR. Traditional approach relies on the use of a multivariate normal distribution, under which the dependence among the profit margins is assumed to be symmetric and captured by linear correlation. Instead of restricting our attention to the traditional multivariate normal assumption, this study employs the copula-based Monte Carlo simulation method to calculate CVaR. The copula-based method provides more flexibility in modeling the dependence between profit margins generated at different production locations in that it allows us to model the marginal distributions and their dependence structure separately. In addition, the copula function can capture non-linear dependence and thus provides a more accurate picture of the relationship among farm profitability in different locations.

Within this context, the aim of this paper is to examine the potential benefits of a downside risk reduction from a geographical diversification strategy in agriculture. Our investigation adds to the work of earlier studies in three important ways. First, to the best of our knowledge, the effectiveness of farm-level geographical diversification in wheat farming has not been investigated or reported in the literature. Second, the paper combines CVaR (the popular risk management criterion) and copulas (a mathematical tool used in finance to model a dependence structure of asset returns) to construct a model for the evaluation of a diversification strategy. This allows us to construct a portfolio of production locations and to evaluate trade-offs in profitability and downside risk without restricting to any specific distribution assumptions. Finally, the study compares the optimization results from the copula-based mean-CVaR model with those from the multivariate-normal mean-CVaR model. The comparison then allows us to observe the effects of relying on the standard assumption of multivariate normality in terms of portfolio optimization results.

The remainder of the paper is organized as followed. Section 2 outlines the model and method. Section 3 is devoted to describing the data. Section 4 presents and discusses the results of copula selection, mean-CVaR optimization, and optimal portfolio allocation. Finally, Section 5 summarizes and concludes.

2. Model and method
In this section, we first present a model for evaluating geographical diversification. Our model is built on the portfolio theory developed by Markowitz (1952) and the CVaR measure. We then discuss the method used to calculate CVaR.

2.1 Model
To evaluate the effects of geographical diversification, we assume that a producer may operate in more than one location, and that his objective is to choose the optimal share of total acres allocated to each production locations. Let w_i be the number of acres allocated to a production location i, where $i = [1, n]$, and W be the total acres dedicated

to wheat production. Given n possible choices of growing location, a share of total acres allocated to production location i, $\alpha_i = w_i/W$, is chosen such that $\sum_{i=1}^{n} \alpha_i = 1$. As a measure of farm profitability, we use the profit margin from production. The profit margin measures the percentage by which a farm's total revenue exceeds its total expenses. The profit margin from location i's production can be stated as:

$$\pi_i = \frac{p_i y_i w_i - c_i w_i}{p_i y_i w_i} = \frac{p_i y_i - c_i}{p_i y_i} \tag{1}$$

where p_i, y_i, and c_i are the location i's price per bushel of wheat, yield of wheat per acre, and cost of production per acre of wheat, respectively. Further, it is assumed that the producer's optimal location decision will depend upon the weighted-average profit margin (expected profit margin), which can then be calculated as:

$$\pi_P = \sum_{i=1}^{n} \alpha_i \pi_i \tag{2}$$

To calculate the optimal resource allocation, an appropriate risk measure must be chosen. The Markowitz's (1952) mean-variance optimization framework uses a standard deviation of the portfolio's return as a risk measure. However, the standard deviation is not an appropriate measure of downside risk, as it punishes both upside and downside deviations equally. An alternative measure of downside risk is VaR. VaR determines the amount of potential loss in a portfolio value (in this case, the weighted-average profit margin) over a given period of time for a particular confidence level. For example, if a farm has a one-year 95 percent VaR of 0.15, there is a 5 percent chance that its profit margin will drop to less than −0.15 (or −15 percent) within any given year. VaR is formally defined as:

$$VaR_\beta(Y) = -\min\left[y \mid Pr(Y \leqslant y) \geqslant \beta\right] \tag{3}$$

where Pr is a probability distribution, β is a given confidence level, and Y is the expected profit margin over a given time horizon. The advantages of VaR are its simplicity and intuitive interpretation. In addition, VaR only penalizes downside risk, and thus addresses the major drawback of the mean-variance optimization approach. Nevertheless, recent research has shown that VaR does not possess the subadditivity property, one of the properties that a risk measure should have (Artzner *et al.*, 1999; Acerbi, 2007). A particularly troubling implication for portfolio optimization is that the VaR of a portfolio of two securities may be greater than that of each individual security, suggesting that diversification should be discouraged (Alexander and Baptista, 2004). Furthermore, VaR is also shown to lead to erroneous results when the data are not normally distributed (Stoica, 2006).

CVaR, introduced by Rockafellar and Uryasev (2000), has been gaining popularity as an alternative to VaR. Similar to VaR, CVaR does not penalize upside gains. CVaR measures the expected loss given that the loss is greater than or equal to VaR (i.e. the average of all losses in the worst $(1-\beta)\%$ cases, where β is a confidence level). It satisfies the four properties of a coherent risk measure: translation invariance, subadditivity, positive homogeneity, and monotonicity. These are the properties that risk measures should have (Artzner *et al.*, 1999). Moreover, CVaR has been found to be a more consistent measure of risk than VaR and generally resulted in more efficient portfolio choices (Rockafellar and Uryasev, 2000; Alexander and Baptista, 2004).

In addition, CVaR is a more appropriate measure than variance because it does not assume normality of distributions of returns of assets. We therefore use CVaR as a measure of risk in this study. Mathematically, CVaR is defined as:

$$CVaR_\beta(Y) = -E\big[Y\,\big|\,Y \leqslant -VaR_\beta(Y)\big] \tag{4}$$

Given the return and risk measures, the producer's portfolio optimization problem can be expressed as:

$$\max_{\alpha_i} \pi_P = \sum_{i=1}^{n} \alpha_i \pi_i = \sum_{i=1}^{n} \alpha_i \left(\frac{p_i y_i - c_i}{p_i y_i}\right) \tag{5}$$

subject to:

$$CVaR_\beta(\alpha_i) \leqslant \phi \tag{6}$$

$$\sum_{i=1}^{n} \alpha_i = 1 \tag{7}$$

where α_i is a share of total acres allocated to production location i, and the CVaR in Equation (6) is set to equal to the parameter ϕ, defined as the target CVaR level. The mean-CVaR efficient frontier can then be derived by solving the above optimization problem for different levels of ϕ. This frontier gives us the maximum portfolio return (maximum expected profit margin) for a given level of CVaR.

2.2 Method
Calculation of CVaR requires knowledge of a cumulative distribution function of portfolio returns, which in turn depends on the joint distribution of returns of all assets included in the portfolio. A traditional approach relies on the use of a multivariate normal distribution (Markowitz, 1952). However, the assumption of normality for agricultural prices and yields has shown to be inconsistent (Goodwin and Ker, 2002; Ramirez *et al.,* 2003). Copulas are an alternative method of modeling joint distributions that has been gaining popularity in financial literature including portfolio analysis (Clemen and Reilly, 1999; Bouyé *et al.*, 2001; Hennessy and Lapan, 2002; Alexander *et al.*, 2006, 2007; Bai and Sun, 2007). The main advantage of the copula approach is that it allows us to specify the marginal distributions of prices and yields (as well as profitability measures) and their dependence structure separately. Even though the copula approach has been used in finance for quite some time, its applications in the agricultural literature are recent (see, e.g. Vedenov, 2008; Zhu *et al.*, 2008; Power *et al.*, 2009; Larsen *et al.*, 2013). This study uses the copula-based Monte Carlo simulation to simulate the distributions of the producer's profit margins and compute the corresponding value of CVaR.

The copula-based Monte Carlo simulation method can be briefly summarized in three steps. In the first step, we construct a joint distribution of variables of interest (i.e. profit margins). According to Sklar's (1959) theorem, the joint distribution $F(x_1, \ldots, x_n)$ can be represented as:

$$F(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n)) \tag{8}$$

where $C: [0, 1]^2 \rightarrow [0, 1]$ is a unique copula function and $F_i(x_i)$ are marginal distributions of variables of interest. This implies that the unknown joint distribution can be constructed by estimating the marginal distribution functions from the historical profit margin data

and selecting a parametric functional form of the copula. In this study, we use kernel smoothing transformation to estimate the marginal density functions of historical profit margins (cf. Goodwin and Ker, 1998; Vedenov, 2008; Power *et al.*, 2009). Regarding the form of copula function, we are looking for the copula that covers lower tail dependence. Possible candidates include Clayton, Rotated Gumbel, and Student's *t*. The most appropriate copula function is selected based on the three information selection criteria: the Akaike Information Criterion (AIC), Hannan-Quinn Information Criterion (HQIC), and Schwarz Information Criterion (SIC). The definitions of the copula functions considered in this study are given in Table I. For comparison, the multivariate normal distribution is also used in the calculation of CVaR.

The selected copula is then combined with a kernel density estimate of marginal distributions of farm-level profit margins to construct the joint distribution of profit margins. In this study, copula parameters are estimated through maximum likelihood estimation method:

$$\hat{\theta} = argmax_\theta \sum_{t=1}^{T} \ln c\Big(\hat{F}_1(x_{1t}), \ldots, \hat{F}_n(x_{nt}); \theta\Big) \tag{9}$$

where θ is the copula parameter, $c(\cdot)$ is a copula density, and $F_i(x_i)$ are the estimated marginal distributions. A more complete overview of copula functions, properties and their applications can be found in Joe (1997), Embrechts *et al.* (2003), Cherubini *et al.* (2004), Patton (2004), McNeil *et al.* (2005) and Nelsen (2006).

In the second step, a series of Monte Carlo draws of the *n*-tuples $(\pi_1, \ldots, \pi_n)$ are generated as follows. First, a random number vector $(u_1, \ldots, u_n)$, whose marginal distribution follow a uniform distribution, is generated from a chosen copula function $C(u_1, \ldots, u_n)$. Second, the random number vector $(\pi_1, \ldots, \pi_n)$ are then obtained by inversely transforming the marginal distribution F_i of a variable π_i; i.e., $(\pi_1, \ldots, \pi_n) = \Big(F_1^{-1}(u_1), \ldots, F_n^{-1}(u_n)\Big)$. In the final step, using the simulated realizations of profit margin for each location, the mean-CVaR efficient frontier is estimated by solving the above optimization problem.

3. Data
To analyze the effectiveness of geographical diversification as a risk management strategy, this research is focussed on the specialized production of winter and spring wheat in three separate production locations (i.e. $n = 3$): the combination of winter and spring wheat provides an efficient way to maximize machinery and planting/harvesting windows based on traditional planting and harvesting windows in three regions. Smith County, Kansas; Bottineau County, North Dakota; and Moore County, Texas. The three

Copula function	Definition
Clayton	$C(u_1, \ldots, u_n \mid \theta) = \big(\sum_{i=1}^n u_i - (n-1)\big)^{-1/\theta}$ with $0 < \theta < \infty$
Rotated Gumbel	$C(u_1, \ldots, u_n \mid \theta) = exp\Big[\Big(-\sum_{j=1}^n \big(-\log(1-u_j)^\theta\big)\Big)^{1/\theta}\Big]$ with $1 \leqslant \theta < \infty$
Student's *t*	$C(u_1, \ldots, u_n \mid v, P) = t_{v,P}\big(t_v^{-1}(u_1), \ldots, t_v^{-1}(u_n)\big)$

**Notes:** t_ν is the standard Student's *t* distribution with degree of freedom ν and $t_{v,P}$ is the CDF of a multivariate Student's *t* vector of dimension n with degree of freedom ν and correlation matrix P

Table I.
Definitions of clayton, Gaussian, rotated Gumbel, and Student's *t* copula functions

geographically distinct areas are chosen based on harvesting windows and distance criteria. The three locations have at least partial overlap in harvesting windows, which allows for machinery sharing (Wolfley, 2008). Accordingly, the costs involved with geographical diversification may be reduced or even eliminated when machinery sharing is possible. Generally in this paper, the state names are used to refer to the farm locations.

Farm level data were gathered from the North Dakota Farm Business Management Association, Kansas Farm Business Management Association, and Texas A&M Extension service. Farm level data covers the period from 2003 to 2012. Annual data for state-level prices were collected from the National Agricultural Statistics Service, and annual data for region-level production operating costs per planted acre are collected from United States Department of Agriculture. The operating cost data covers the period from 1975 to 2013. These data are used because they are the most consistent available data for each farm location. The operating costs include seed, fertilizer, chemicals, custom operations, fuel, lube, electricity, repairs, labor, general farm overhead, taxes, insurance, other variable expenses, and interest on operating inputs. The historical yields, prices and operating costs are used to calculate the profit margin for each location. For the purpose of this analysis, any forms of government payments are not considered in the calculation of profit margins. Figure 1 illustrates historical farm-level profit margins in each location. The profit margins in Kansas appear to be the most stable but in most years, the profit margin in at least one location moves in the opposite direction to that in the other locations. For example, the profit margins in North Dakota fell sharply from the year 2003 to 2005. However, Kansas's and Texas's profit margins during that same time period either remained stable or increased. This suggests that geographical diversification may be a viable risk management strategy for wheat producers.

Table II reports summary statistics on profit margins. Texas has the highest average profit margin (0.63), whereas North Dakota has the lowest average profit margin (0.57). Conversely, North Dakota has the widest range of profit margins, ranging from 0.29 to 0.83, while Kansas's profit margins range from 0.46 to 0.69. The highest profit margin for Texas is 0.84 and the lowest is 0.40. The minimum and maximum margins for North Dakota and Texas reveal that both locations have the potential for high profitability but also the potential for extremely low profitability. Kansas, on the other hand, does not display extremely high or extremely low profit
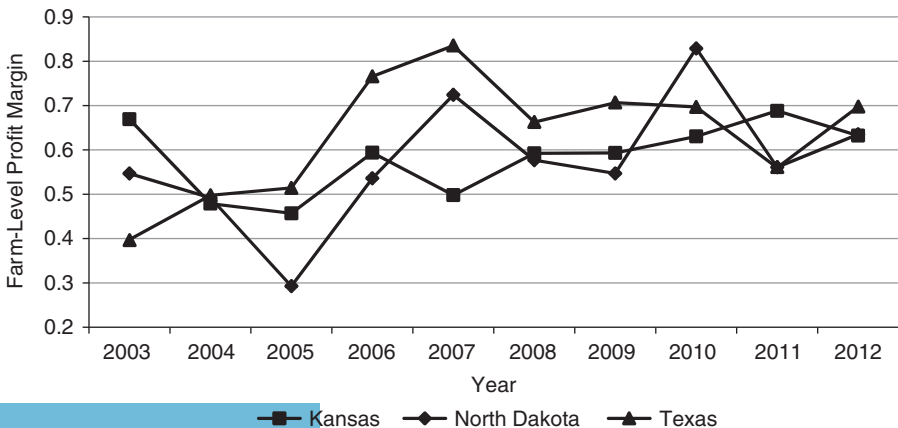
Figure 1.
Historical farm-level profit margins in Kansas, North Dakota, and Texas

margins. As expected, North Dakota displays the highest standard deviation whereas Kansas exhibits the lowest standard deviation. Thus, from both North Dakota and Texas perspective, locating production in Kansas could provide some risk reduction based solely on the visual inspection of the summary statistics.

Included in the summary statistics (Table II) is the diagnostic analysis for annual profit margins. Regarding the higher moments of the data, Kansas has the lowest absolute value of skewness and the lowest kurtosis, while North Dakota has the highest absolute value skewness and the lowest kurtosis. A normal distribution has skewness of 0 and kurtosis of 3. Kansas and Texas have considerably higher skewness (in absolute term) and lower kurtosis than the normal distribution, suggesting that the distributions of their profit margins are asymmetric, and non-normal. The normality tests (Shapiro-Wilk tests) fails to reject the hypothesis that the data are not normally distributed. Testing the farm level data are problematic based on the low sample size. The skewness and kurtosis values indicate the existence of heavy tails which justifies the additional effort of using copulas to model the joint distribution of profit margins from the three locations. Further, the Ljung-Box tests (Q-statistics) show that there is no serial correlation in all series. Moreover, both the Q^2 and Lagrange Multiplier statistics for the ARCH effects display no autocorrelations in the squared returns for all series, implying that heteroskedasticity is not present in profit margin series. This indicates that the observations are approximately independent and identically distributed. In addition, the augmented Dickey-Fuller and Kwiatkowski-Phillips-Schmidt-Shin tests show that all series are stationary, suggesting that their mean and covariance do not change over time. Accordingly, the marginal distribution function for each location can be estimated directly from the historical profit margin data.

To investigate the comovement of profit margins for each location pair, we first calculate Pearson's correlation coefficient, Spearman's rho coefficient, and Kendall's tau coefficient. These are the three most commonly used measure of degree of dependence. The results are presented in Table III. The Pearson's correlation coefficients range from −0.076 (Kansas and Texas) to 0.55 (North Dakota and Texas). The Spearman's rho coefficients range from −0.05 (Kansas and Texas) to 0.49 (North Dakota and Texas). The Kendall's tau coefficients range from −0.007 (Kansas and Texas) to 0.33 (North Dakota and Texas). The differing degrees of dependence suggests that geographical diversification across different production locations may significantly benefit producers in enhancing farm's profitability with lower downside risk. Results of detailed analysis on geographical diversification benefits follow.

	Kansas	North Dakota	Texas
Mean	0.5833	0.5740	0.6334
Maximum	0.6882	0.8289	0.8353
Minimum	0.4569	0.2926	0.3977
SD	0.0799	0.1415	0.1359
Skewness	−0.4059	−0.1143	−0.2893
Kurtosis	1.8580	3.4037	2.0598
SW test	0.9111	0.9057	0.9567
p-value	0.2885	0.2139	0.7482

Notes: SW test is a Shapiro-Wilk test for normality

Table II.
Summary statistics for farm-level profit margins

4. Results

We now apply our model and method to our data set. The results of the producer's portfolio-optimization problem are provided in this section. We first present the copula selection results. We then discuss mean-CVaR optimization and optimal portfolio allocation results.

4.1 Copula selection results

To prepare for copula modeling, we first estimate the marginal density functions of historical profit margins of the three locations using kernel density estimation. These estimated marginal distributions are then used in the estimation of the copula functions.

Table IV presents the goodness-of-fit statistics. As mentioned above, we use AIC, HQIC and SIC to select the most appropriate copula model. Based on the three criteria, Clayton copula is found to be the best copula model, followed by Rotated Gumbel and Student's t copulas. Accordingly, the copula function we use is the multivariate Clayton copula. The estimated value of Clayton copula parameter, θ, is 0.281. We then apply the copula-based Monte Carlo simulation method, and obtain 1,000 simulations of profit margins for each location from the Clayton copula. For comparison, the traditional multivariate normal distribution is also used to generate another set of data using the Monte Carlo simulation. In this case, the profit margins are assumed to follow a multivariate

	Kansas	North Dakota	Texas
Pearson's correlation coefficient			
Kansas	1.0000	0.4123	−0.0765
North Dakota		1.0000	0.5515
Texas			1.0000
Spearman's ρ			
Kansas	1.0000	0.3697	−0.0545
North Dakota		1.0000	0.4909
Texas			1.0000
Kendall's τ			
Kansas	1.0000	0.2444	−0.0667
North Dakota		1.0000	0.3333
Texas			1.0000

Table III. Degree of dependence of farm-level profit margins across locations (measured by pearson's correlation coefficient, Spearman's rho, and Kendall's τ)

Copula function	AIC	HQIC	SIC
Clayton	−1.5355	0.1021	−0.9528
Rotated Gumbel	−1.4577	0.1799	−0.8751
Student's t	0.0237	3.2988	1.1890

Notes: The best-fit model is the one with the minimum AIC, HQIC and SIC values. AIC, Akaike Information Criterion: $AIC = ((2T)/(T-k-1))k - 2\ln(LL)$; HQIC, Hannan-Quinn Information Criterion: $HQIC = 2\ln(\ln(T))k - 2\ln(LL)$; and SIC, Schwarz Information Criterion: $SIC = \ln(T)k - 2\ln(LL)$, where T is the number of observations, k is the number of copula parameters estimated, and LL is the value of maximum likelihood

Table IV. Copula goodness-of-fit statistics

normal distribution (i.e. individual profit margin distributions and their dependence are assumed normal). The two sets of simulated data (generated from the Clayton copula and multivariate normal) are used for the following geographical diversification analysis.

4.2 Mean-CVaR optimization results
We maximize the producer's portfolio value subject to the CVaR constraint. Table V presents examples of optimal portfolios (portfolios on the mean-CVaR efficient frontiers) at different confidence levels (90, 95, and 99 percent) for the copula-based and traditional multivariate-normal portfolios. By definition, the reported values of CVaR indicate the magnitude of losses, so the negative values imply the magnitude of gains. For instance, 95 percent CVaR of 0.10 indicates that the average profit margin in the worst 5 percent of the cases is 10 percent, whereas 95 percent CVaR of −0.10 reveals that the average profit margin in the worst 5 percent of the cases is 10 percent. To compare the values of CVaR under different distribution assumptions, the portfolios, for each level of confidence, are chosen such that they have the same expected profit margins. For all confidence levels, the copula-based portfolio yields considerably higher value of CVaR than the multivariate-normal portfolio. Overall, Table V suggests that if the data are correctly described by the Clayton copula, the producer will underestimate the minimum level of downside risk (measured by CVaR) for a given level of expected profit margin. The underestimation of risk under the assumption of multivariate normal distribution is clearly shown in Figure 2.

Figure 2 depicts the mean-CVaR efficient frontiers for the 95 percent confidence level for the copula-based and traditional multivariate-normal portfolios. The Clayton copula accounts for lower tail dependence, whereas the multivariate normal distribution assumes that the coefficient of the lower tail dependence is zero. As can be seen from the figure, the copula-based frontier lies greatly below the multivariate-normal frontier. This indicates that the copula-based frontier accounts for the downside risk more than the multivariate-normal one. In other words, the multivariate-normal portfolio seems to underestimate the CVaR for a given level of expected profit margin compared with the copula-based portfolio. This occurs because the multivariate-normal model ignores the comovement in the lower tail of the joint distribution. When lower tail dependence is present, the downside risk should be expected to be higher. Hence, relying on the traditional multivariate normal assumption may be considered a less conservative portfolio-optimization approach and thus appears inappropriate for the producers who are concerned with the extreme losses of farm profitability.
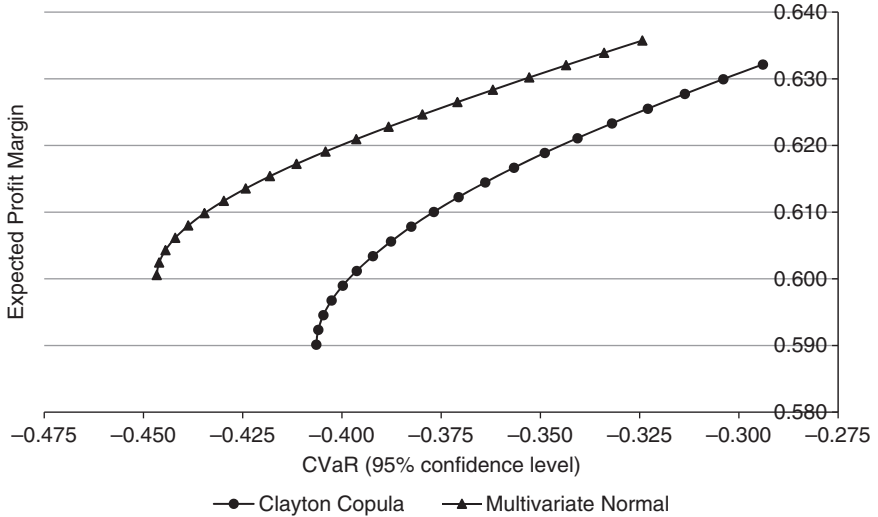
The efficient frontiers in Figure 3 demonstrate how reduction in downside risk is made possible by geographical diversification. The figure shows where the specific production locations (Kansas, North Dakota and Texas) located relative to the frontiers.

	90% Expected profit margin	CVaR	95% Expected profit margin	CVaR	99% Expected profit margin	CVaR
Clayton copula	0.609	−0.414	0.608	−0.383	0.615	−0.294
Multivariate normal	0.609	−0.476	0.608	−0.454	0.615	−0.386

Notes: The Conditional Value-at-Risk (CVaR) is reported as the magnitude of losses. This negative value of CVaR indicates the magnitude of gains
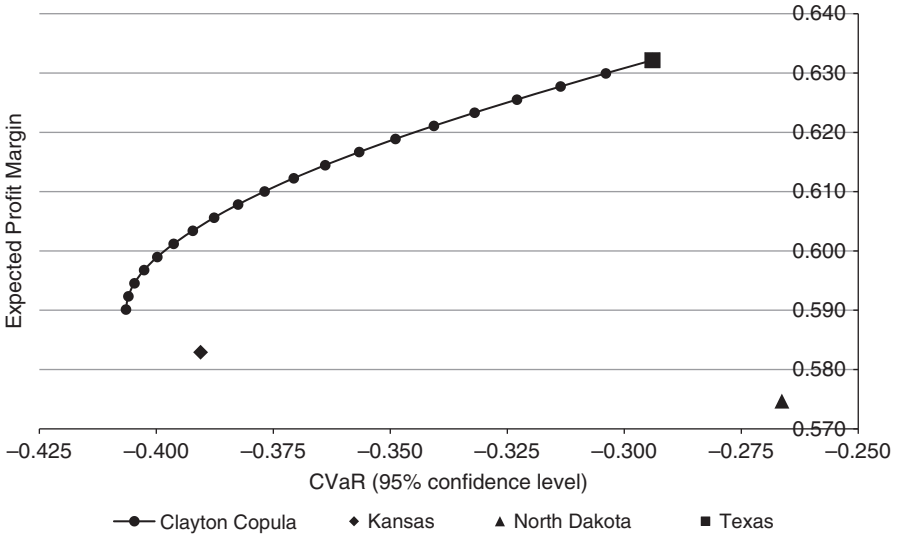
Table V.
Examples of optimal portfolio at 90, 95, and 99 percent confidence levels for copula-based and traditional multivariate-normal portfolios

Notes: The Conditional Value-at-Risk (CVaR) is reported as the magnitude of losses.
Therefore, the negative value of CVaR indicates the magnitude of gains

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Therefore, the negative value of CVaR indicates the magnitude of gains

For producers currently operating in Kansas and North Dakota, geographical diversification will enhance their profitability because both locations are located below the frontiers. The only location that is on the frontiers is Texas. Thus, at the particular level of downside risk faced by the producers in Texas, geographical diversification will not improve their profitability. However, Texas is located at the top right of the

curves and thus producers in Texas face the maximum CVaR. Given the shape of the frontiers, the benefits for producers in Texas from reducing farm profitability a little bit through diversification (moving down the curve) are relatively large in terms of downside risk reduction. Specifically, based on the copula-based frontier, wheat producers in Texas can increase their average profit margin in the worst 5 percent of the cases from −21.6 to 25.3 percent by allocating part of total production (about 15 percent) to Kansas. This requires reducing their expected profit margins from 63.22 to 62.50 percent (a reduction of just 0.72-percentage points, comparing to a downside risk reduction of 3.7 percentage points). This result is not surprising because the average historical profit margin in Kansas is 5.01 lower than that of Texas, whereas the standard deviation of historical profit margins in Kansas is 5.60-percentage points lower than that in Texas. Thus, the benefit from risk reduction is larger than the loss in the expected return. However, it should be noted that the rates of risk-return trade-offs are not constant along the frontiers. Unlike at the high levels of expected profit margin, at the low levels of expected profit margin (at the seemingly vertical part of the curves), producers can increase their expected profit margin without facing much higher level of risk through production reallocation (i.e. by reducing production share allocated to North Dakota and increasing shares allocated to Texas and Kansas). This is not unexpected because even though standard deviation of historical profit margins in Texas is lower than that of North Dakota, it is higher than that of Kansas (while historical average profit margin in North Dakota is lower than that of Texas and Kansas). In fact, the benefits from producing in North Dakota come almost entirely from the low correlations of profit margins across locations. Therefore, the benefits from moving part of production away from North Dakota to Texas and Kansas are relatively large in terms of the increased expected profits. In the next section, we discuss these portfolio optimization results in terms of the optimal share of acreage allocation among the three specific production areas.

4.3 Optimal portfolio allocation results

We now analyze the optimal percentage of acreage allocation among the three production locations. Figures 4 and 5 illustrate the efficient allocations corresponding to different points of the mean-CVaR efficient frontiers (for 95 and 99 percent confidence levels) for the copula-based and traditional multivariate-normal portfolios. As expected, the optimal share allocated to each location differs based on the expected profit margin (and risk). In the maximum risk case, the optimal choice is to allocate the total acres to the location with the highest expected profit margin (in this case, Texas). The minimum CVaR portfolios involve the producers operating in all the three locations with the largest proportion of land allocated to Kansas. At the low to intermediate levels of expected profit margin (and risk), it is optimal to allocate more than 50 percent of total acres to Kansas, and less than 10 percent to North Dakota. At the intermediate to high levels of profitability, production should be operated mostly in Texas and none should be operated in North Dakota. Comparing the results from the copula-based and traditional multivariate-normal portfolios, the two main allocation differences occur with: first, North Dakota share is zero for all optimal allocations and second, Kansas share of total acres becoming the major part of the efficient portfolio allocation later for the case of multivariate normal distribution. This is expected because, in contrast to the Clayton copula, the multivariate normal distribution assumes zero tail dependence and thus values less the potential risk reduction benefits from diversifying to the locations with lower level of profitability. These results illustrate once again the impact of not accounting
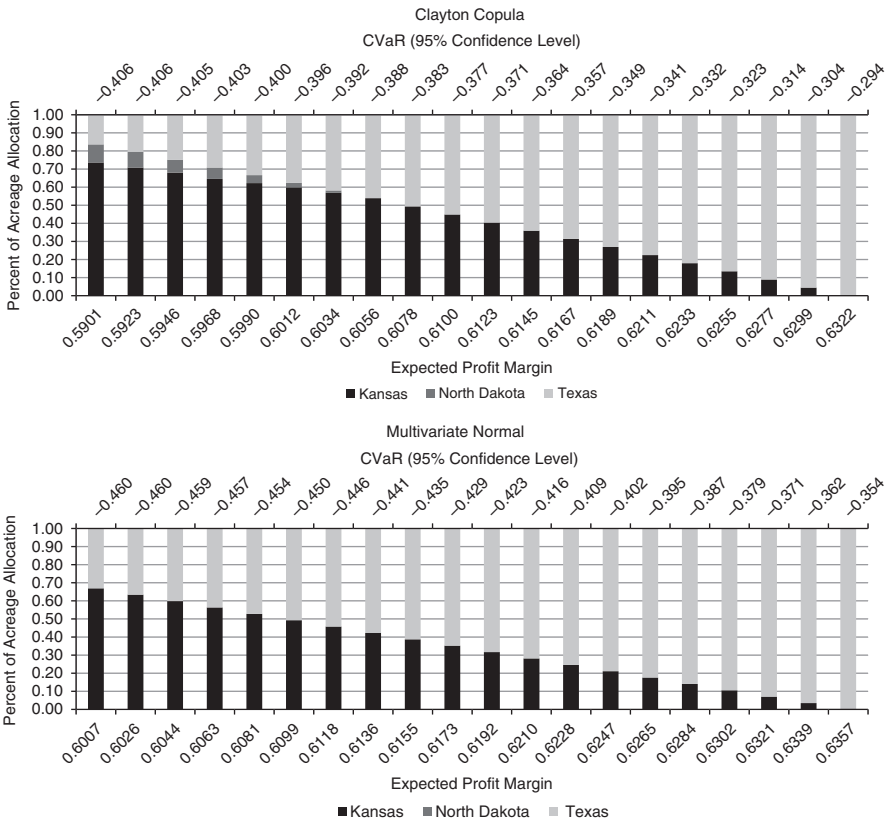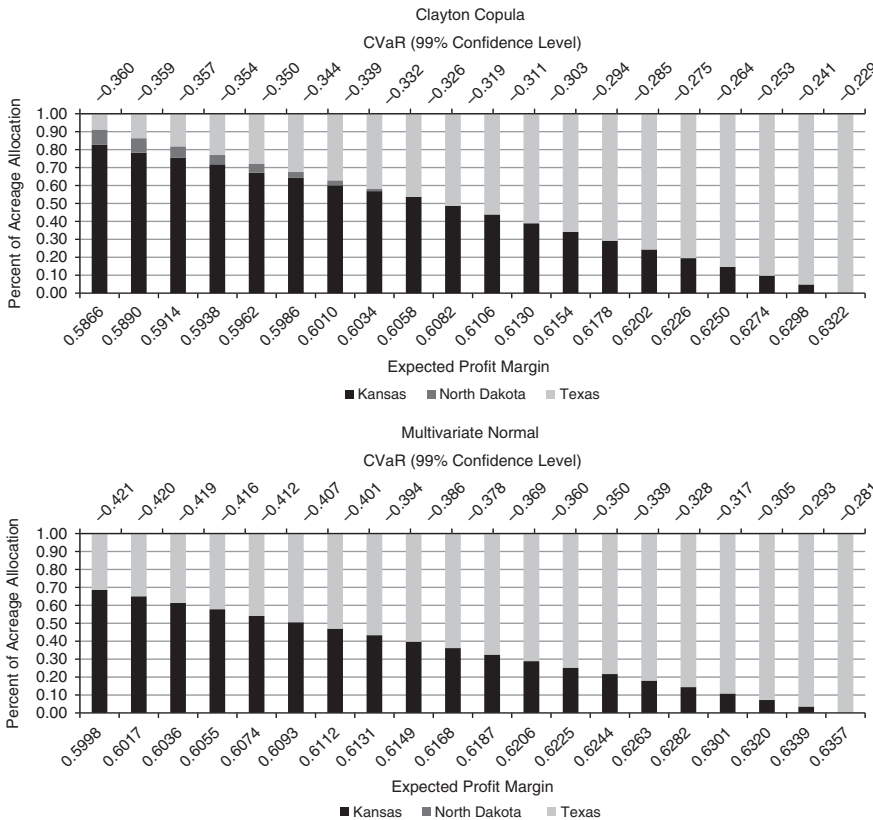
Figure 4.
Percent of acreage
allocation among
the three
production regions
(at 95 percent
confidence level)
for copula-based
and traditional
multivariate-normal
portfolios

for the lower tail dependence. By ignoring the comovement in the lower tail of the joint distribution, efficient allocations at the intermediate to high levels of profitability would consist of more acreage in Texas and would not consider Kansas in reducing risk. Nevertheless, both copula-based and traditional multivariate-normal portfolios agree that geographical diversification seems to be an effective risk management tool for wheat producers.

5. Conclusion
The paper presents a model for evaluating the effectiveness of a geographical diversification in agriculture: the copula-based mean-CVaR model. The model is built on the portfolio theory, CVaR measure, and copula approach.

Not only does the model allow for a broad range of different dependence structures and thus allow for a more flexible joint distribution (i.e. non-normal multivariate distribution), but it also employs CVaR, one of the recent most popular risk management criteria and the recent most appropriate measure of downside risk, as a measure of risk. The proposed model is then applied to the producer's acreage allocation problem (equivalently, portfolio-optimization problem) to examine the potential benefits of risk reduction from a geographical diversification strategy in US wheat farming. Specifically, the portfolio

Figure 5.
Percent of acreage
allocation among the
three production
regions
(at 99 percent
confidence level) for
copula-based
and traditional
multivariate-normal
portfolios

consists of three specific production locations: Smith County, Kansas; Bottineau County, North Dakota; and Moore County, Texas. To the best of our knowledge, this is the first attempt to rigorously analyze whether geographical diversification is a viable risk management strategy for wheat producers utilizing several tools from modern finance theory.

An analysis of farm-level historical profit margins for the three production locations shows evidence against the assumption of multivariate normality. A Clayton copula is chosen to describe lower tail dependence of profit margins and to construct a joint distribution of profit margins using a copula-based Monte Carlo simulation method. For comparison, the traditional multivariate normal distribution is also used in the construction of a joint distribution. For each distribution assumption, a producer's optimal portfolio (as well as the corresponding efficient frontier) is constructed by maximizing the expected profit margin (or the weighted-average profit margin) for a given level of CVaR. The mean-CVaR optimization results show that reduction in downside risk is made possible for wheat producers in Kansas and North Dakota by geographical diversification because both locations are located below the frontiers. The only production location that gains the least from diversifying geographically is Texas in that it is located on the efficient frontiers. However, given that Texas is located at the most risky part of the curves, the benefits from reducing farm profitability a little

bit through diversification are relatively large in terms of a downside risk reduction. In terms of the optimal share of acreage allocation, the optimal share allocated to each location differs based on the producer's attitude toward the risk and return. Overall, both copula-based and traditional multivariate-normal portfolios agree that a geographical diversification strategy is a legitimate risk management strategy.

The study results also illustrate the benefits of incorporating lower tail dependence into the construction of the joint distribution. By relying on the assumption of multivariate normality and ignoring the existence of lower tail dependence, the producers can run the risk of underestimating the minimum level of downside risk for a given level of expected profit margin. Therefore, the copula-based method may be considered a more conservative approach in analyzing the effectiveness of a geographical diversification strategy. The implication of these results should encourage researchers to move beyond the standard assumptions of linear correlation and normality.

Based on our farm-level copula-based Monte Carlo simulation model, geographical diversification enhances farm profitability from a farm's profit margin perspective. By splitting production among the three locations, wheat producers could possibly increase their profitability given the same level of downside risk. However, the results of this study do not take into consideration the costs that could be incurred when producing in more than one state. Thus, one potentially fruitful avenue of future research would be to account for cost-related factors in geographical diversification. Future studies can also apply the proposed model to other agricultural commodities. These extensions would add to the current understanding of the effects of geographical diversification in agriculture.

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